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Higgs-Dilaton Cosmology: An Effective Field Theory Approach

ΜΕΤΑΠΤΥΧΙΑΚΗ ΔΙΠΛΩΜΑΤΙΚΗ ΕΡΓΑΣΙΑ

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Ευχαριστίες

Θα ήθελα να εκφράσω τις θερμές μου ευχαριστίες στους καθηγητές μου Κώστα Φαράκο και Misha Shaposhnikov για την εμπιστοσύνη που μου έχουν δείξει και για την καθοδήγηση και τις συμβουλές τους. Ευχαριστώ επίσης τον μεταδιδακτορικό ερευνητή Javier Rubio, συνεπιβλέποντα της εργασίας αυτής, για την πολύ εποικοδομητική συνεργασία που είχαμε τους περασμένους μήνες.

Η παρούσα εργασία δε θα μπορούσε να πραγματοποιηθεί χωρίς την υποστήριξη της οικογένειας και των φίλων μου, στους οποίους και την αφιερώνω.

Σύνοψη

Το κοσμολογικό μοντέλο Higgs-dilaton προβλέπει μία πληθωριστική περίοδο στο Πρώιμο Σύμπαν, καθώς και μία περίοδο στη μετέπειτα εξέλιξή του, όπου η επιταχυνόμενη διαστολή οφείλεται σε χρονικά εξελισσόμενη Σκοτεινή Ενέργεια. Στο συγκεκριμένο μοντέλο οι δύο αυτές ανεξάρτητες εποχές συνδέονται μέσω μίας μη τετριμμένης σχέσης ανάμεσα στις ανομοιογένειες n_s του αρχέγονου φάσματος και της καταστατικής εξίσωσης ω της Σκοτεινής Ενέργειας. Σκοπός της παρούσας εργασίας είναι η μελέτη του μοντέλου Higgs-dilaton με τεχνικές της Κβαντικής Θεωρίας Πεδίου. Λαμβάνοντας υπ' όψιν μας ότι το κενό της θεωρίας μεταβάλλεται με το χρόνο, υπολογίζουμε την ενέργεια στην οποία η θεωρία παύει να είναι αυτοσυνεπής. Δείχνουμε ότι αυτό το "κατώφλι" ενέργειας, το οποίο εξαρτάται από τα πεδία που είναι παρόντα στη θεωρία, είναι παραμετρικά μεγαλύτερο από τις χαρακτηριστικές ενεργειακές κλίμακες καθ' όλη την εξέλιξη του Σύμπαντος. Τέλος, διατυπώνουμε τις υποθέσεις που μας επιτρέπουν να εκτιμήσουμε την επίδραση των κβαντικών διορθώσεων με συστηματικό τρόπο, και να δείξουμε ότι η σχέση ανάμεσα στα n_s και ω παραμένει αμετάβλητη.

Abstract

The Higgs-dilaton model is able to describe simultaneously an inflationary period in the early Universe and a dark energy (DE) dominated stage responsible for the present day acceleration. It also leads to a relation between the tilt ns of the scalar spectrum perturbations and the equation of state ω of DE. We study the self-consistency of this model from an effective field theory point of view. Taking into account the influence of the dynamical background fields, we determine the effective cut-off of the theory, which turns out to be parametrically larger than all the relevant energy scales during the history of the Universe from inflation till present time. We formulate the set of assumptions that allow to estimate an amplitude of quantum corrections in a systematic way and show that the connection between ns and ω remains unaltered if these assumptions are satisfied.

Contents

Περίληψη

Το πρότυπο της Μεγάλης Έκρηξης (Hot Big Bang model) είναι σε θέση να εξηγήσει μία πληθώρα παρατηρησιακών δεδομένων. Παρ΄όλ' αυτά, υπάρχουν αρκετά ερωτήματα τα οποία δε μπορούν να βρουν απάντηση στα πλαίσιά του συγκεκριμένου προτύπου.

Τα ερωτήματα τα οποία σχετίζονται με την χωρική επιπεδότητα, την ομοιογένεια και ισοτροπία του Σύμπαντος καθώς και την έλλειψη μαγνητικών μονοπόλων απαντώνται στα πλαίσια πληθωριστικών θεωριών. Σύμφωνα με αυτές, το Σύμπαν κατά τις πρώτες στιγμές της δημιουργίας του διεστάλλει με εκθετικό ρυθμό. Στα απλούστερα πληθωριστικά μοντέλα, η διαστολή οφείλεται στην κυριαρχία της δυναμικής έναντι της κινητικής ενέργειας ενός βαθμωτού πεδίου, του inflaton.

Σύμφωνα με το μοντέλο Higgs-inflation, το ρόλο του inflaton μπορεί να παίξει το πεδίο Higgs, το οποίο κατέχει ξεχωριστή θέση στο Καθιερωμένο Πρότυπο των Στοιχειωδών Σωματιδίων. Για να υπάρξει συμφωνία με τις διακυμάνσεις της θερμοκρασίας στην Κοσμική Μικροκυματική Ακτινοβολία Υποβάθρου, πρέπει το Higgs να είναι συζευγμένο με τη βαρύτητα και η τιμή της αντίστοιχης σταθεράς σύζευξης ξ_h να είναι μεγάλη. Η μελέτη του μοντέλου αυτού απλοποιείται αισθητά αν πραγματοποιήσουμε έναν σύμμορφο μετασχηματισμό ώστε το τμήμα που περιγράφει τις βαρυτικές αλληλεπιδράσεις να αποκτήσει τη συνηθισμένη μορφή της Λαγκρατζιανής Einstein- Hilbert. Η θεωρία που προκύπτει όμως είναι μη-πολυωνυμική και συνεπώς μη-ανακανονικοποιήσιμη, ακόμα και χωρίς την παρουσία της βαρύτητας. Για το λόγο αυτό πρέπει να θεωρείται ως μία ενεργός θεωρία πεδίου, η οποία παραμένει αυτοσυνεπής μέχρι μία συγκεκριμένη ενεργειακή κλίμακα, το κατώφλι Λ. Το σύνηθες κριτήριο για τον προσδιορισμό του κατωφλιού είναι η παραβίαση της μοναδιακότητας σε σκεδάσεις που

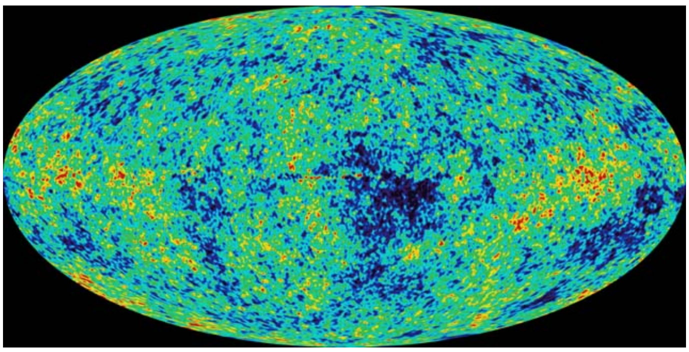


Figure 1: Ανομοιογένειες στην Κοσμική Μικροκυματική Ακτινοβολία Υποβάθρου.

λαμβάνουν χώρα σε υψηλές ενέργειες.

Λεπτομερής ανάλυση φανέρωσε ότι το κατώφλι της θεωρίας εξαρτάται από την αναμενόμενη τιμή στο κενό του πεδίου Higgs, η οποία μεταβάλλεται καθώς το σύστημα μεταβαίνει από τον πληθωρισμό στις μετέπειτα φάσεις της εξέλιξής του. Η τιμή του κατωφλιού βρέθηκε παραμετρικά υψηλότερη από τις χαρακτηριστικές ενεργειακές κλίμακες καθ' όλη την ιστορία του Σύμπαντος. Ως αποτέλεσμα, η θεωρία όπου το πεδίο Higgs είναι υπεύθυνο για την εκθετική διαστολή του Σύμπαντος παραμένει μία αυτοσυνεπής ενεργός θεωρία πεδίου.

Το σενάριο Higgs-inflation ενσωματώθηκε σε ένα ευρύτερο πλαίσιο, το κοσμολογικό μοντέλο Higgs-dilaton. Χαρακτηριστικό γνώρισμα αυτού είναι η αναλλοιώτητα ως προς μετασχηματισμούς κλίμακας

$$\Phi^i(x) \to \sigma^{d_{\Phi^i}}\Phi^i(\sigma x) \ , \tag{1}$$

όπου $\Phi^i(x)$ είναι τα πεδία της θεωρίας, σ μία τυχαία σταθερά και d_{Φ^i} η διάσταση κλίμακας των πεδίων. Για να επιτύχουμε συμμετρία ως προς τους μετασχηματισμούς (1) και παράλληλα η θεωρία που προκύπτει να είναι φαινομενολογικά βιώσιμη, είναι αναπόφευκτη η εισαγωγή ενός νέου βαθμωτού σωματιδίου, του dilaton, του οποίου η αναμενόμενη τιμή στο κενό απαιτούμε να είναι μη μηδενική . Σαν αποτέλεσμα, όλες οι κλίμακες επάγονται μέσω της παραβίασης της συμμετρίας ως προς τους μετασχηματισμούς κλίμακας , και το πεδίο dilaton, που είναι το μποζόνιο Goldstone της παραβιασμένης συμμετρίας, παραμένει άμαζο και η αλληλεπίδραση του με την ύλη είναι πολύ ασθενής.

Παρόλο που η συμμετρία (1) δε μας επιτρέπει να εισάγουμε ρητά έναν όρο Κοσμολογικής Σταθεράς στη θεωρία, τον αντίστοιχο ρόλο παίζει η παράμετρος β που περιγράφει την αλληλεπίδραση του dilaton με τον εαυτό του. Αν αυτή η παράμετρος επιλεγεί να είναι μηδέν, μία ελαφριά τροποποίηση της Γενικής Σχετικότητας οδηγεί στην εμφάνιση χρονικά εξαρτώμενης Σκοτεινής Ενέργειας, η οποία είναι εξ' ολοκλήρου υπεύθυνη για την διαστολή του Σύμπαντος που παρατηρούμε. Πιο συγκεκριμένα, αν επιβάλλουμε στην ορίζουσα g της μετρικής του χωρόχρονου το δεσμό $g = -1$, τότε αυτόματα εμφανίζεται ένα εκθετικό δυναμικό για το dilaton, κάνοντάς το κατάλληλο υποψήφιο για πεδίο quintessence.

Το μοντέλο Higgs-dilaton περιγράφει μία περίοδο πληθωρισμού στο Πρώιμο Σύμπαν καθώς και μια περίοδο κυριαρχίας Σκοτεινής Ενέργειας, στην οποία οφείλεται η διαστολή που παρατηρούμε σήμερα. Είναι σε θέση να συνδέσει αυτές τις δύο ανεξάρτητες περιόδους, κάτι που κάνει τη θεωρία κατά κάποιο τρόπο μοναδική.

Ο σκοπός της παρούσας εργασίας είναι η μελέτη του συγκεκριμένου μοντέλου από τη σκοπία της Κβαντικής Θεωρίας Πεδίου. Αρχικά υπολογίζουμε τα διάφορα ενεργειακά κατώφλια που σχετίζονται με αλληλεπιδράσεις ανάμεσα σε βαθμωτά πεδία, τη βαρύτητα, τα διανυσματικά πεδία και τα φερμιόνια. Δείχνουμε ότι η τιμή του κατώτερου από αυτά, είναι παραμετρικά υψηλότερη από τις χαρακτηριστικές ενεργειακές κλίμακες σε όλες τις φάσεις της εξέλιξης του Σύμπαντος. Έπειτα μελετάμε τις κβαντικές διορθώσεις. Εφ' όσον η θεωρία είναι μη-ανακανονικοποιήσιμη, απαιτείται η εισαγωγή ενός άπειρου αριθμού όρων για την απορρόφηση των υπεριωδών απειρισμών που εμφανίζονται. Δείχνουμε ότι αν το σχήμα ανακανονικοποίησης σέβεται τις ακριβείς και κατά προσέγγιση συμμετρίες της κλασσικής θεωρίας, οι προβλέψεις του μοντέλου παραμένουν αναλλοίωτες.

Chapter 1

Introduction

The shortcomings of the hot big bang model can be solved in an elegant way if we assume that the Universe underwent an inflationary period in its early stages. The easiest way for this scenario to be realized is by a scalar field slowly rolling towards the minimum of its potential [1].

In [2] it was shown that this scalar - the inflaton - can be the Higgs field of the Standard Model (SM) with the mass lying in the interval where the SM can be considered a consistent effective theory up to the inflationary scale. More precisely, if the Higgs boson is non-minimally coupled to gravity and the value of the corresponding constant ξ_h is large, it is able to describe an inflationary period followed by the hot Big Bang. The implications of this scenario have been extensively studied in the literature [4, 7, 6, 5, 11, 3, 10, 12, 13, 14, 18, 15, 19, 8, 9, 16, 17, 20]. Earlier studies of non-minimally coupled fields in the context of inflation can be also found in [21, 22, 23].

When the theory with non-minimal coupling of the Higgs field to Ricci scalar is rewritten in the Einstein frame, it becomes essentially non-polynomial and thus non-renormalizable, even if the gravity part is dropped off. Therefore, it should be understood as an effective field theory valid up to a certain cut-off energy scale. The usual criterion for determining the cut-off of the theory is based on the violation of tree level unitarity in high-energy scattering processes.

The tree-level scattering amplitudes above the *electroweak vacuum state* appear to hit the unitarity bound at energies $\Lambda \sim M_P/\xi_h$ [8, 9, 16, 17]. At that scale perturbation theory breaks down. Whether the theory is self-consistent and just enters into the non-perturbative strong-coupling regime or requires an ultraviolet completion at higher energies remains unknown. In spite of the fact that Λ is smaller than the inflationary scale $M_P/\sqrt{\xi_h}$, the Higgs inflation scenario is self-consistent. As it has been shown in [24] (see also [25]), the cutoff of the theory depends on the Higgs background, making the theory weakly coupled for all the relevant energy scales in the evolution of the Universe. In other words, the SM with large non-minimal coupling to gravity represents a viable effective field theory for inflation, reheating, and subsequent evolution of the Universe.

The Higgs inflation scenario was incorporated into a larger framework, the Higgs-dilaton model [27, 28]. The key element of this extension is scale-invariance (SI). No dimensional parameters, such as masses, are allowed to appear in the action. All scales are induced by the spontaneous breaking of SI. This is achieved by the introduction of a new scalar degree of freedom, the dilaton χ. As a consequence, the physical dilaton becomes the Goldstone boson of the broken symmetry, remaining exactly massless. The coupling of the dilaton to matter is weak and takes place only through derivative couplings, not contradicting therefore any 5th force experimental bounds [30].

The dilatation symmetry does not allow adding a cosmological constant to the action. Still, the cosmological constant problem is not solved, as it reappears, being now related to the dilaton self-coupling β (see below) in the Jordan frame [27]. However, even if β is chosen to be zero (or required to vanish due to some unknown yet reason), a slight modification of general relativity (GR) leads to dynamical DE in accordance with observations of accelerating Universe. Namely, the scale-invariant Unimodular Gravity (UG) gives rise to a "run-away" dark energy potential for the dilaton [27], which plays the role of a quintessence field. The strength of such a potential is determined by an integration constant that appears in the Einstein equations of motion due to the unimodular constraint $g = -1$, where g is the determinant of the metric in the Jordan frame.

The Higgs-dilaton scenario was shown be able to explain simultaneously the early and late Universe in a consistent way. The common origin of the inflationary and dark energy dominated stages allowed to derive extra bounds on the initial inflationary conditions, as well as potentially testable relations between the early and late Universe observables [28].

Our purpose here is to study, following [24], the self-consistency of the Higgs-dilaton model by adopting an effective field theory point of view. We will estimate the field-dependent cut-offs associated to the different interactions among scalars fields, gravity, vector bosons and fermions. We will identify the lowest cut-off as a function of the value of the background fields and show that its value is higher than the energy scales describing the Universe during different epochs. The issue concerning quantum corrections generated by the loop expansion is also addressed. Since the model is non-renormalizable, an infinite number of counter-terms have to be added in order to absorb the divergences. We adopt the "minimal setup" for construction of a theory with all ultraviolet divergences removed by an infinite number of counter-terms. In this procedure no new degrees of freedom are introduced, and renormalization procedure keeps the exact and approximative symmetries of the classical action intact. We will show that the relations connecting the inflationary and the dark energy domination periods, hold in the presence of quantum corrections within this approach.

The structure of the thesis is as follows. In Section 2 we briefly review the Higgs-dilaton model. In Section 3 we calculate in detail the cut-off of the theory in the Jordan frame and compare it with the other relevant energy scales in the evolution of the Universe. In Section 4 we propose a "minimal setup" which removes all the divergences and discuss the sensitivity of inflationary and the dark energy predictions to radiative corrections. Section 5 contains the conclusions.

Chapter 2

Higgs-dilaton cosmology

We start by reviewing the main results of [27, 28], where the Higgs-dilaton model was proposed and studied in detail. The two main ingredients of the theory are outlined below. The first one is the requirement of invariance of the action under scale transformations, which leads to the absence of any dimensional parameters. If we denote with $\Phi(x)$ the fields of a theory, with σ^{d_Φ} their scaling dimensions and σ an arbitrary constant, scale transformations can be written as global transformations[1]

$$g_{\mu\nu}(x) \to g_{\mu\nu}(\sigma x) \ , \quad \Phi(x) \to \sigma^{d_\Phi}\Phi(\sigma x) \ . \tag{2.1}$$

To achieve invariance under the above transformations, we let masses and dimensional couplings to be induced dynamically by replacing them with a field. The simplest choice would be to use the SM Higgs field for the appearance of the scales. However, this would contradict experimental constraints since the excitations of the Higgs field become massless, decouple completely from the other SM particles and interact only with the gravitational field [29].

The next simplest option is to introduce a new scalar singlet under the SM gauge group, the dilaton χ. The coupling between the new singlet and all the SM particles with exception of the Higgs boson is forbidden by quantum numbers. The corresponding Lagrangian is

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{1}{2}(\xi_\chi \chi^2 + 2\xi_h \phi^\dagger \phi)R + \mathscr{L}_{\mathrm{SM}[\lambda\to 0]} - \frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi,\phi) \ , \tag{2.2}$$

where ϕ is the SM Higgs field doublet, R the scalar curvature, and $\xi_\chi \sim 10^{-3}$, $\xi_h \sim 10^5$ are respectively the non-minimal couplings of the dilaton and Higgs field to gravity, whose values are determined from CMB observations [27, 28]. The term $\mathscr{L}_{\mathrm{SM}[\lambda\to 0]}$ is the

[1]For a theory invariant under all diffeomorphisms, scale transformations can be rewritten as local transformations:

$$g_{\mu\nu}(x) \to \sigma^{-2}g_{\mu\nu}(x) \ , \quad \Phi(x) \to \sigma^{d_\Phi}\Phi(x) \ .$$

SM Lagrangian without the Higgs potential, which in the present scale-invariant theory becomes

$$V(\chi,\phi) = \lambda \left(\phi^\dagger\phi - \frac{\alpha}{2\lambda}\chi^2\right)^2 + \beta\chi^4 \ , \tag{2.3}$$

with $\lambda > 0$ being the Higgs self-coupling. In order for this theory to be phenomenologically viable, we have to demand that there exist symmetry-breaking ground states with non-vanishing background values for the dilaton and the Higgs field[2]. These are

$$\bar{h}^2 = \frac{\alpha}{\lambda}\bar{\chi}^2 + \frac{\xi_h}{\lambda}R \ , \quad \text{with} \quad R = \frac{4\beta\lambda}{\lambda\xi_\chi + \alpha\xi_h}\bar{\chi}^2 \ , \tag{2.4}$$

where the bar denotes the background value of the fields. All scales induced will be proportional to the non-zero background value of the dilaton field. For example, we can identify the physical SM Higgs mass as

$$m_H^2 = 2\alpha M_P^2 \frac{(1+6\xi_\chi) + \frac{\alpha}{\lambda}(1+6\xi_h)}{(1+6\xi_\chi)\xi_\chi + \frac{\alpha}{\lambda}(1+6\xi_h)\xi_h} + \mathcal{O}(\beta) \ . \tag{2.5}$$

Note also that depending on the value of the dilaton self-coupling β, there exist ground states for the model that correspond to flat ($\beta = 0$), deSitter ($\beta > 0$) or anti-deSitter ($\beta < 0$) spacetimes, with cosmological constant

$$\Lambda = \frac{1}{4}M_P^2 R = \frac{\beta M_P^4}{(\xi_\chi + \frac{\alpha}{\lambda}\xi_h)^2 + 4\frac{\beta}{\lambda}\xi_h^2} \ . \tag{2.6}$$

It is important to notice that physical observables, corresponding to dimensionless ratios between scales or masses, are independent of the particular value of the background field $\bar{\chi}$. The parameters of the model must be fixed in such a way that the correct ratios between the different scales are reproduced. From Eq. (2.5) it is clear that in order to account for the difference between the electroweak and Planck scales, we must require $\alpha \lll 1$ [3]. Similarly, the hierarchy between the cosmological and the electroweak scales, cf. Eq. (2.6), implies $\beta \lll \alpha$. Both hierarchies remain unexplained within the theory under consideration.

The second ingredient is the replacement of GR with UG. Unimodular Gravity is a particular case of a much more general set of theories invariant under the group of transverse diffeomorphisms TDiff. TDiff theories generically contain an extra scalar degree of freedom on top of the massless graviton (see for example [39]). UG reduces the dynamical components of the metric by requiring the metric determinant g to take some fixed constant value, conventionally $|g| = 1$. Unimodular gravity is then invariant only

[2]If $\bar{\chi} = 0$ the Higgs field is massless, and if $\bar{h} = 0$ there is no electroweak symmetry breaking.

[3]One could argue that in order to reproduce the correct ratio between m_H and M_P we could choose a large value for the non-minimal coupling ξ_χ of the dilaton to the Ricci scalar, instead of requiring $\alpha \lll 1$. This choise however is not phenomenologically acceptable, since $\xi_\chi \sim 10^{-3}$, in order for the predictions of the model to be in agreement with the CMB observations.

under volume-preserving diffeomorphisms. It can be shown [27] that the solutions of a theory subject to the unimodular constraint $\hat{g} \equiv \det(\hat{g}_{\mu\nu}) = -1$

$$\mathscr{L}_{\rm UG} = \mathscr{L}[\hat{g}_{\mu\nu}, \partial\hat{g}_{\mu\nu}, \Phi, \partial\Phi] \ , \tag{2.7}$$

coincide with those ones obtained from a theory which is invariant under the full group of diffeomorphisms (Diff) but with modified action

$$\frac{\mathscr{L}_{\rm Diff}}{\sqrt{-g}} = \mathscr{L}[g_{\mu\nu}, \partial g_{\mu\nu}, \Phi, \partial\Phi] + \Lambda_0 \ . \tag{2.8}$$

From the point of view of UG, the parameter Λ_0 is just a conserved quantity associated to the unimodular constraint and it should not be understood as a cosmological constant. As was shown in [27] (see also below), for $\Lambda_0 > 0$, due to the non-minimal couplings of the fields to gravity, an exponential "run-away" potential for the physical dilaton appears automatically, making it responsible for dark energy. These types of potential have been already considered in pioneering works on quintessence in [33, 34, 35].

Since the two formulations are equivalent, we stick to the Diff invariant language. Expressing the theory resulting from the combination of the above ideas in the unitary gauge in the SM sector $\phi^T = (0, h/\sqrt{2})$ we have

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{1}{2}(\xi_\chi\chi^2 + \xi_h h^2)R - \frac{1}{2}(\partial\chi)^2 - \frac{1}{2}(\partial h)^2 - U(\chi, h) \ , \tag{2.9}$$

where the potential is now given by

$$U(\chi, h) \equiv V(\chi, h) + \Lambda_0 = \frac{\lambda}{4}\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2 + \beta\chi^4 + \Lambda_0 \ . \tag{2.10}$$

Let us notice at this point an important property of the previous Lagrangian that will turn out to be essential in further developments. As discussed above, in order to reproduce the hierarchy of scales, the parameters α and β must satisfy $\beta \lll \alpha \lll 1$. This, together with the small value of the non-minimal coupling $\xi_\chi \ll 1$ between the dilaton field and gravity, gives rise to an approximate shift symmetry for the dilaton field at the classical level, $\chi \to \chi + \text{const}$. This symmetry would be exact for $\alpha = \beta = \xi_\chi = 0$. This fact has important consequences for analysis of quantum effects. We will come back to it in Section 4, where the "minimal" renormalization procedure will be formulated.

Let us discuss the phenomenological implications of the model. For this end it is more convenient to rewrite the Lagrangian in the Einstein frame, where the gravity part takes the usual Einstein-Hilbert form. This is achieved by performing a conformal redefinition of the metric [42]

$$\tilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \tag{2.11}$$

with conformal factor $\Omega^2 = M_P^{-2}(\xi_\chi\chi^2 + \xi_h h^2)$. Using

$$\sqrt{-g} = \Omega^{-4}\sqrt{\tilde{g}} \qquad \text{and} \qquad R = \Omega^2\left(\tilde{R} + 6\tilde{\Box}\ln\Omega - 6\tilde{g}_{\mu\nu}\partial_\mu\ln\Omega\partial_\nu\ln\Omega\right) \ , \tag{2.12}$$

we get

$$\frac{\mathscr{L}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2}\tilde{R} - \frac{1}{2}\tilde{K}(\chi,h) - \tilde{U}(\chi,h) \ , \tag{2.13}$$

where the non-canonical kinetic term $\tilde{K}(\chi,h)$ in the basis $(\phi^1,\phi^2) = (\chi,h)$ is given by

$$\tilde{K}(\chi,h) = \frac{\kappa_{ij}}{\Omega^2}\tilde{g}^{\mu\nu}\partial_\mu\phi^i\partial_\nu\phi^j \ , \quad \text{with} \quad \kappa_{ij} = \left(\delta_{ij} + \frac{3}{2}M_P^2\frac{\partial_i\Omega^2\partial_j\Omega^2}{\Omega^2}\right) \ . \tag{2.14}$$

Note that the kinetic term could be cast into canonical form if and only if the two minimal couplings of the scalar fields to gravity were equal $\xi_\chi = \xi_h$, which is not consistent with observations [28]. The potential is given by

$$\tilde{U}(\chi,h) = \frac{U(\chi,h)}{\Omega^4} \equiv \frac{M_P^4}{(\xi_\chi\chi^2 + \xi_h h^2)^2}\left[\frac{\lambda}{4}\left(h^2 - \frac{\alpha}{\lambda}\chi^2\right)^2 + \beta\chi^4 + \Lambda_0\right] \ . \tag{2.15}$$

The action (2.9) does not contain any dimensional parameters, except the one proportional to Λ_0. The divergency of the dilatational Noether's current associated to scale transformations will be therefore proportional to Λ_0. As shown in [28], the whole inflationary period takes place inside the domain of field space where the contribution of Λ_0 is negligible. We will refer to this domain as the "scale invariant region" and assume that it is maintained even when quantum corrections are taken into account. This will be justified in Section 4. As a consequence, the current is approximately conserved and there exists a set of variables (r,θ') in terms of which the scale transformation acts as a shift on r only. Since this transformation is a symmetry of the theory when $\Lambda_0 = 0$, r will appear only through the part of the potential which is proportional to Λ_0 and terms with derivatives. We can also choose θ' in a way that the kinetic terms are diagonal. We define[4]

$$\begin{aligned} r &= \frac{M_P}{2\gamma}\log\left[\frac{(1+6\xi_\chi)\chi^2 + (1+6\xi_h)h^2}{M_P^2}\right] \ , \\ |\theta'| &= \frac{M_P}{a}\tanh^{-1}\left[\sqrt{\frac{(1-\varsigma)(1+6\xi_\chi)\chi^2}{(1+6\xi_\chi)\chi^2 + (1+6\xi_h)h^2}}\right] \ , \end{aligned} \tag{2.16}$$

with

$$\gamma = \sqrt{\frac{\xi_\chi}{1+6\xi_\chi}} \ , \quad a = \sqrt{\frac{\xi_\chi(1-\varsigma)}{\varsigma}} \ , \quad \varsigma = \frac{(1+6\xi_h)\xi_\chi}{(1+6\xi_\chi)\xi_h} \ . \tag{2.17}$$

The physical interpretation of these variables is straightforward. They are simply adequately rescaled polar variables in the (h,χ) plane. The angular variable is therefore periodic and defined in the compact interval $\theta' \in [-\theta_0, \theta_0]$, where θ_0 is given below in eq. (2.19). In terms of these variables the Lagrangian (2.13) takes a very simple form

$$\frac{\mathscr{L}}{\sqrt{-\tilde{g}}} = \frac{M_P^2}{2}\tilde{R} - \frac{\varsigma\cosh^2[a\theta/M_P]}{2}(\partial r)^2 - \frac{1}{2}(\partial\theta)^2 - \tilde{U}(\theta) - \tilde{U}_{\Lambda_0}(r,\theta) \ , \tag{2.18}$$

[4]In this work the definition of the angular variable θ is slightly different than in [28]. The parametrization we use shifts the minimum of the potential and therefore simplifies the study of the (p)reheating period of the Universe.

where

$$\theta = \theta_0 - |\theta'| \ , \quad \text{with} \quad \theta_0 = \frac{M_P}{a} \tanh^{-1}\left[\sqrt{1-\varsigma}\right] \ . \tag{2.19}$$

The potential is naturally divided into a scale-invariant and a scale-invariance breaking parts, given respectively by

$$\begin{aligned} \tilde{U}(\theta) = & \frac{\lambda M_P^4}{4\xi_h^2(1-\varsigma)^2}\left(c - \sigma\cosh^2[a\theta/M_P]\right)^2 \\ & + \beta M_P^4\left(\frac{1+6\xi_h}{\xi_h - \xi_\chi}\right)^2\left(1-\cosh^2[a\theta/M_P]\right)^2 \ , \end{aligned} \tag{2.20}$$

and

$$\tilde{U}_{\Lambda_0}(r,\theta) = \frac{\Lambda_0}{\gamma^2}\varsigma^2\cosh^4[a\theta/M_P]e^{-4\gamma r/M_P} \ , \tag{2.21}$$

where we have defined

$$c = 1 + \frac{\alpha}{\lambda}\frac{1+6\xi_h}{1+6\xi_\chi} \ , \quad \sigma = \varsigma + \frac{\alpha}{\lambda}\frac{1+6\xi_h}{1+6\xi_\chi} \ . \tag{2.22}$$

Inflation takes place between $\theta_i = M_P/\alpha\tanh^{-1}\sqrt{1-\varsigma}$ and $\theta_{\rm f} \simeq 0$, where, as can be seen in Fig.2.1 , the potential $\tilde{U}$ has the same shape as the one in the Higgs inflation. For later convenience we present the analytical expressions for the amplitude and spectral tilt of the scalar perturbations produced during inflation in the case $\beta \lll 1$. At order $\mathcal{O}(\xi_\chi, 1/\xi_h, 1/N^*)$ we have

$$P_\zeta(k_0) \simeq \frac{\lambda\sinh^2[4\xi_\chi N^*]}{1152\pi^2\xi_\chi^2\xi_h^2} \ , \tag{2.23}$$

$$n_s(k_0) - 1 \simeq -8\xi_\chi\coth(4\xi_\chi N^*) \ , \tag{2.24}$$

where N^* denotes the number of e-folds between the moment at which the pivot scale $k_0/a_0 = 0.002\ \mathrm{Mpc}^{-1}$ exited the horizon and the end of inflation. In the parameter space where $1 < 4\xi_\chi N^* \ll 4N^*$, the expression for the tilt becomes linear in ξ_χ and is given by

$$n_s(k_0) - 1 \simeq -8\xi_\chi \ . \tag{2.25}$$

An interesting cosmological phenomenology arises with the peculiar choice $\beta = 0$[5]. In this case, the DE dominated period in the late Universe depends only on the dilaton field r, and non-trivial relations between the inflationary and the DE domination periods can be established. Once the system reaches the minimum of the potential, i.e. for $\tanh^2[a\theta/M_P] \simeq 1-\sigma$, a “run-away” potential for the dilaton appears, making it suitable for playing the role of quintessence. Let us assume that the scale-invariance breaking term $\tilde{U}_{\Lambda_0}(r,\theta)$ is negligible during the radiation and matter dominated stages, but responsible for the present accelerated expansion of the Universe. In this case it is possible to write

[5]Some arguments in favour of the $\beta = 0$ case can be found in Ref. [54, 28, 39].

the following relation between the equation of state parameter ω_r of the r field and its relative abundance Ω_r [43]

$$1+\omega_r = \frac{16\gamma^2}{3}\left[\frac{1}{\sqrt{\Omega_r}} - \frac{1}{2}\left(\frac{1}{\Omega_r}-1\right)\log\frac{1+\sqrt{\Omega_r}}{1-\sqrt{\Omega_r}}\right]^2 \ . \tag{2.26}$$

Since dark energy is due to the quintessence field r only, we can identify ω_r as $\omega_{\rm DE}$. For the observed value of $\Omega_r \simeq 0.74$, the above expression yields

$$1+\omega_{\rm DE} = \frac{8}{3}\frac{\xi_\chi}{1+6\xi_\chi} \ . \tag{2.27}$$

From equations (2.25) and (2.27) follows that the deviation n_s of the scalar spectrum from the scale-invariant one is proportional to the deviation of dark energy from a cosmological constant[6] [28]

$$n_s - 1 \simeq -3(1+\omega_{\rm DE}), \quad \text{for} \quad \frac{2}{3N^*} < 1+\omega_{\rm DE} \ll 1 \ . \tag{2.29}$$

It is important to notice that the above condition is a non-trivial prediction of Higgs-dilaton cosmology, relating two a priori completely independent periods in the history of the Universe. This has interesting consequences from an observational point of view[7] and makes the Higgs-dilaton scenario rather unique. We will be back to equation (2.29) in Section 4, where we will show that it still holds even in the presence of quantum corrections within the "minimal setup".

[6]Outside this region of parameter space, the relation connecting n_s to $\omega_{\rm DE}$ is somehow more complicated and given by

$$n_s - 1 \simeq -\frac{12(1+\omega_{\rm DE})}{4-9(1+\omega_{\rm DE})}\coth\left[\frac{6N^*(1+\omega_{\rm DE})}{4-9(1+\omega_{\rm DE})}\right] \ . \tag{2.28}$$

[7]Similar consistency relations relating the rate of change of the equation of state parameter $w(a) = w_0 + w_a(1-a)$ with the logarithmic running of the scalar tilt can be also derived, cf. [28]. The practical relevance of those consistence conditions is however much more limited than that of Eq. (2.29), given the small value of the running of the scalar tilt in Higgs-driven scenarios.

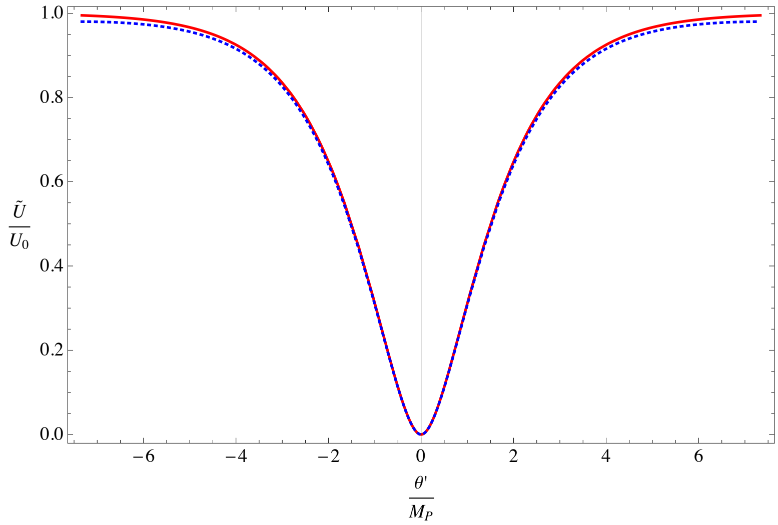


Figure 2.1: Comparison between the Higgs-dilaton inflationary potential (blue dashed line) obtained from (2.20) in the scale-invariant region and the corresponding one for the Higgs Inflation model (red continuous line). The amplitudes are normalized to the asymptotic value $U_0 \simeq \frac{\lambda M_P^4}{4\xi_h^2}$.

Chapter 3

The dynamical cut-off scale

Following [24], we now turn to the determination of the energy domain where the Higgs-dilaton model can be considered as a predictive effective field theory. This domain is bounded from above by the field-dependent cut-off $\Lambda(\Phi)$, i.e. the energy where perturbative tree-level unitarity is violated [44]. At energies above that scale, the theory becomes strongly-coupled and the standard perturbative methods fail.

In order to determine this (background dependent) energy scale, two related methods, listed below, can be used.

1. Expand the generic fields of the theory around their background values

$$\Phi(\mathbf{x}, t) = \bar{\Phi} + \delta\Phi(\mathbf{x}, t) \ , \tag{3.1}$$

   such that all sorts of higher-dimensional non-renormalizable operators of the form

$$c_n \frac{\mathcal{O}_n(\delta\Phi)}{[\Lambda(\bar{\Phi})]^{n-4}} \ , \tag{3.2}$$

   with $c_n \sim \mathcal{O}(1)$ appear in the resulting action. These operators are suppressed by appropriate powers of the field-dependent coefficient $\Lambda(\bar{\Phi})$, which can be identified as the cut-off of the theory. This procedure gives us only a lower estimate of the cut-off, since it does not take into account the possible cancelations that occur between the different scattering diagrams.

2. Calculate at which energy each of the N-particle scattering amplitudes hit the unitarity bound. The cut-off will then be the lowest of these scales.

In what follows we will apply these two methods to determine the effective cut-off of the theory. We will start by applying the prescription 1) to determine the cut-off associated with the gravitational and scalar interactions. The cut-off associated to the gauge and fermionic sectors will be obtained via the prescription 2).

3.1 Cut-off in the scalar gravity sector

We choose to work in the original Jordan frame where the Higgs and dilaton fields are non-minimally coupled to gravity. Expanding these fields around a static scalar background

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu} \ , \qquad \chi = \bar{\chi} + \delta\chi \ , \qquad h = \bar{h} + \delta h \ , \tag{3.3}$$

we obtain the following kinetic term for the quadratic Lagrangian of the gravity and scalar sectors

$$\begin{aligned} \mathscr{K}_2^{\rm G+S} = & \frac{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}{8} \left(\delta g^{\mu\nu} \Box \delta g_{\mu\nu} + 2\partial_\nu \delta g^{\mu\nu} \partial^\rho \delta g_{\mu\rho} - 2\partial_\nu \delta g^{\mu\nu} \partial_\mu \delta g - \delta g \Box \delta g\right) \\ & -\frac{1}{2}(\partial\delta\chi)^2 - \frac{1}{2}(\partial\delta h)^2 + (\xi_\chi \bar{\chi}\delta\chi + \xi_h \bar{h}\delta h)(\partial_\lambda \partial_\rho \delta g^{\lambda\rho} - \Box\delta g) \ . \end{aligned} \tag{3.4}$$

The leading higher-order non-renormalizable operators obtained in this way are given by

$$\xi_\chi (\delta\chi)^2 \Box \delta g \ , \quad \xi_h (\delta h)^2 \Box \delta g \ . \tag{3.5}$$

Note that these operators are written in terms of quantum excitations with non-diagonal kinetic terms. In order to properly identify the cut-off the theory, we should determine the normal modes that diagonalize the quadratic Lagrangian (3.4). After doing so and using the equations of motion to eliminate artificial degrees of freedom, we find that the metric perturbations depend on the scalar fields perturbations, a fact that is implicit in the Lagrangian (3.4). The gravitational part of the action (3.4) can be cast into canonical form in terms of a new metric perturbation $\delta\hat{g}_{\mu\nu}$ given by

$$\delta\hat{g}_{\mu\nu} = \frac{1}{\sqrt{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}} \left[(\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2)\delta g_{\mu\nu} + 2\bar{g}_{\mu\nu}(\xi_\chi \bar{\chi}\delta\chi + \xi_h \bar{h}\delta h)\right] \ . \tag{3.6}$$

The cut-off scale associated to purely gravitational interactions becomes in this way the effective Planck scale in the Jordan frame

$$\Lambda_P^2 = \xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2 \ . \tag{3.7}$$

The remaining non-diagonal kinetic term for the scalar fields is given in compact matrix notation by

$$\mathscr{K}_2^{\rm S} = -\frac{1}{2}\bar{\kappa}_{ij} \partial_\mu \delta\phi^i \partial^\mu \delta\phi^j \ , \tag{3.8}$$

where $\bar{\kappa}_{ij}$ is given by (2.14), but now depends only on the background values of the fields, i.e.

$$\bar{\kappa}_{ij} = \frac{1}{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2} \begin{pmatrix} \xi_\chi \bar{\chi}^2(1+6\xi_\chi) + \xi_h \bar{h}^2 & 6\xi_\chi \bar{\chi} \xi_h \bar{h} \\ 6\xi_\chi \bar{\chi} \xi_h \bar{h} & \xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2(1+6\xi_h) \end{pmatrix} \ . \tag{3.9}$$

In order to diagonalize the above expression we make use of the following set of variables

$$\begin{aligned} \delta\hat{\chi} &= \sqrt{\frac{\xi_\chi \bar{\chi}^2(1+6\xi_\chi) + \xi_h \bar{h}^2(1+6\xi_h)}{(\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2)(\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2)}} \left(\xi_\chi \bar{\chi}\delta\chi + \xi_h \bar{h}\delta h\right) \ , \\ \delta\hat{h} &= \frac{1}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h^2 \bar{h}^2}} \left(-\xi_h \bar{h}\delta\chi + \xi_\chi \bar{\chi}\delta h\right) \ . \end{aligned} \tag{3.10}$$

Before going on let us note here that this is precisely the change of variables (up to an appropriate rescaling with the conformal factor Ω) needed to diagonalize the kinetic terms for the scalar perturbations in the Einstein frame. To see this, it is enough to start from (2.14) and expand the fields around their background values $\phi^i \to \bar{\phi}^i + \delta\phi^i$. Keeping the terms with the lowest power in the excitations, $\tilde{K} = \bar{\Omega}^{-2}\bar{\kappa}_{ij}\partial_\mu\delta\phi^i\partial^\mu\delta\phi^j + \mathcal{O}(\delta\phi^3)$, it is straightforward to show that the previous expression can be diagonalized in terms of

$$\delta\hat{\chi} = \bar{\Omega}^{-1}\sqrt{\frac{\xi_\chi\bar{\chi}^2(1+6\xi_\chi)+\xi_h\bar{h}^2(1+6\xi_h)}{(\xi_\chi^2\bar{\chi}^2+\xi_h^2\bar{h}^2)(\xi_\chi\bar{\chi}^2+\xi_h\bar{h}^2)}}\left(\xi_\chi\bar{\chi}\delta\chi+\xi_h\bar{h}\delta h\right) \ ,$$
$$\delta\hat{h} = \bar{\Omega}^{-1}\frac{1}{\sqrt{\xi_\chi^2\bar{\chi}^2+\xi_h^2\bar{h}^2}}\left(-\xi_h\bar{h}\delta\chi+\xi_\chi\bar{\chi}\delta h\right) \ .$$

The effective cut-off of the theory at a given value of the background fields will be the lowest of the scales suppressing the operators in eq. (3.5). Written in terms of the canonically normalized variables (3.6) and (3.10) these operators read

$$\frac{1}{\Lambda_1}(\delta\hat{h})^2\Box\delta\hat{g} \ , \quad \frac{1}{\Lambda_2}(\delta\hat{\chi})^2\Box\delta\hat{g} \ , \quad \frac{1}{\Lambda_3}(\delta\hat{\chi})(\delta\hat{h})\Box\delta\hat{g} \ , \tag{3.11}$$

where the different cut-off scales are given by

$$\Lambda_1 = \frac{\xi_\chi^2\bar{\chi}^2+\xi_h^2\bar{h}^2}{\xi_\chi\xi_h\sqrt{\xi_\chi\bar{\chi}^2+\xi_h\bar{h}^2}} \ , \tag{3.12}$$
$$\Lambda_2 = \frac{(\xi_\chi^2\bar{\chi}^2+\xi_h^2\bar{h}^2)(\xi_\chi\bar{\chi}^2(1+6\xi_\chi)+\xi_h\bar{h}^2(1+6\xi_h))}{(\xi_\chi^3\bar{\chi}^2+\xi_h^3\bar{h}^2)\sqrt{\xi_\chi\bar{\chi}^2+\xi_h\bar{h}^2}} \ , \tag{3.13}$$
$$\Lambda_3 = \frac{(\xi_\chi^2\bar{\chi}^2+\xi_h^2\bar{h}^2)(\xi_\chi\bar{\chi}^2(1+6\xi_\chi)+\xi_h\bar{h}^2(1+6\xi_h))}{\xi_\chi\bar{\chi}\xi_h\bar{h}\,|\xi_h-\xi_\chi|\,\sqrt{\xi_\chi\bar{\chi}^2+\xi_h\bar{h}^2}} \ . \tag{3.14}$$

3.2 Cut-off in the gauge and fermionic sectors

Let us move now to the cut-off associated with the gauge sector. Since we are working in the unitary gauge for SM fields, in order to determine the cut-off associated to the gauge sector of the theory, it is sufficient to look at the tree-level scattering of non-abelian vector fields with longitudinal polarization. It is well known that in the SM the "good" high energy behaviour of these processes is a result of cancellations that occur when we take into account the interactions of the gauge bosons with the excitations δh of the Higgs field [45, 46].

In our case, even though purely gauge interactions remain unchanged, the graphs involving the Higgs field excitations are modified due to the non-canonical kinetic term. This changes the pattern of the cancellations that occur in the standard Higgs mechanism, altering therefore the asymptotic behaviour of these processes and lowering the energy scale where this part of the theory becomes strongly coupled.

To illustrate how this happens, let us consider the $W_L W_L \to W_L W_L$ scattering in the $s-$channel. The relevant part of the Lagrangian is

$$g m_W W_\mu^+ W^{-\mu} \delta h \ , \tag{3.15}$$

where $m_W \sim g\bar{h}$. After diagonalizing the kinetic term for the scalar fields with the change of variables (3.10), the above expression becomes

$$g' m_W W_\mu^+ W^{-\mu} \delta\hat{h} + g'' m_W W_\mu^+ W^{-\mu} \delta\hat{\chi} \ , \tag{3.16}$$

where the effective coupling constants g' and g'' are given by

$$\begin{aligned} g' &= g \frac{\xi_\chi \bar{\chi}}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h \bar{h}^2}} \ , \\ g'' &= g \frac{\xi_h \bar{h}}{\sqrt{\xi_\chi^2 \bar{\chi}^2 + \xi_h \bar{h}^2}} \sqrt{\frac{\xi_\chi \bar{\chi}^2 + \xi_h \bar{h}^2}{\xi_\chi \bar{\chi}^2 (1+6\xi_\chi) + \xi_h \bar{h}^2 (1+6\xi_h)}} \ . \end{aligned} \tag{3.17}$$

From the requirement of tree unitarity of the S-matrix, it is straightforward to show that the scattering amplitude of this interaction hits the perturbative unitarity bound at energy

$$\Lambda_G \simeq \sqrt{\frac{\xi_\chi \bar{\chi}^2 (1+6\xi_\chi) + \xi_h \bar{h}^2 (1+6\xi_h)}{6\xi_h^2}} \ . \tag{3.18}$$

In the limiting cases, the above expression simplifies to

$$\Lambda_G \simeq \begin{cases} \bar{h} & \text{for } \xi_\chi \bar{\chi}^2 \ll \xi_h \bar{h}^2 \ , \\ \frac{\sqrt{\xi_\chi}\bar{\chi}}{\xi_h} & \text{for } \xi_\chi \bar{\chi}^2 \gg \xi_h \bar{h}^2 \ , \end{cases} \tag{3.19}$$

in agreement with the Higgs inflation model.

To identify the cut-off of the fermionic part of the Higgs-dilaton model, we consider the chirality non-conserving process $\bar{f} f \to W_L W_L$. This interaction receives contributions from diagrams with γ and Z exchange ($s-$channel) and from a diagram with fermion exchange ($t-$channel). In the asymptotic high-energy limit, the total amplitude of these graphs grows linearly with the energy at the center of mass. Once again, the $s-$channel diagram with the Higgs excitations unitarizes the associated amplitude [47, 48, 49]. Following therefore the same steps as in the calculation of the gauge cut-off, we find that this part of the theory enters into the strong-coupling regime at

$$\Lambda_F \simeq y^{-1} \frac{\xi_\chi \bar{\chi}^2 (1+6\xi_\chi) + \xi_h \bar{h}^2 (1+6\xi_h)}{6\xi_h^2 \bar{h}} \ , \tag{3.20}$$

where y is the Yukawa coupling constant. The above cut-off is higher than the one for the SM gauge interactions (3.18) during the whole evolution of the Universe.

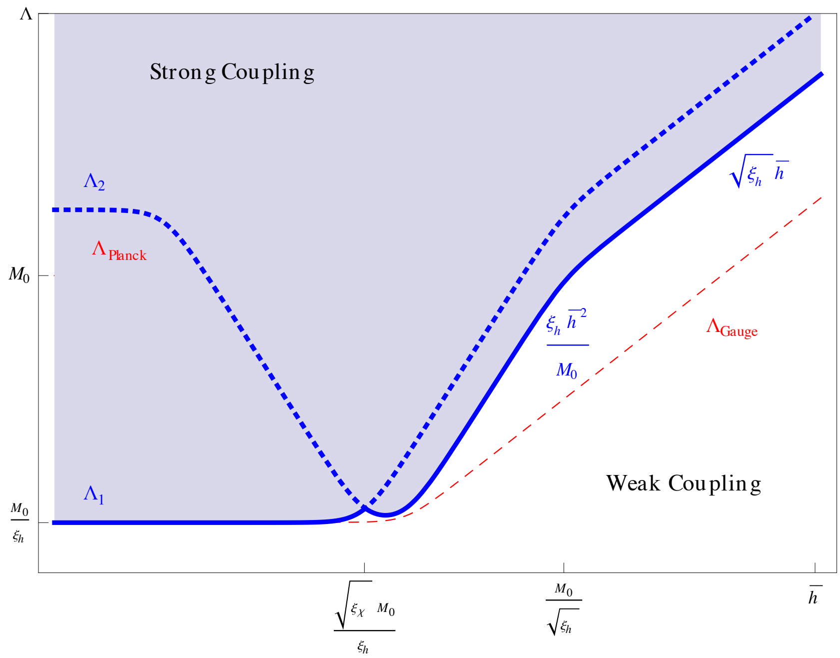


Figure 3.1: Dependence of the different cut-off scales for a fixed value of the dilaton field $\bar{\chi}$ as a function of the Higgs field $\bar{h}$ in the Jordan frame. The cut-off (3.14) is parametrically above the other energy scales (Λ_1, Λ_2, Λ_P, Λ_G and Λ_F) during the whole history and it is therefore not included in the figure. The effective field theory description of scalar fields is applicable for typical energies below the thick blue solid line, which correspond to the minimum of the scalar cut-off scales at a given field value. This is given by Λ_2 and Λ_1 in the scalar sector, for large and small Higgs values respectively. The red solid line correspond to the gravitational cut-off (3.7), while the red dashed one corresponds to the gauge cut-off (3.18). They coincide with the effective scalar sector cut-off for large and small Higgs values respectively. The scale M_0 is defined as $M_0 = \sqrt{\xi_h}\chi$ and corresponds to the value of the effective Planck mass at low energies.

3.3 Comparison with early universe energy scales

In this section we compare the cut-offs found above with the characteristic energy scales in the different periods during the evolution of the Universe. If the typical momentum involved in the different processes is sufficiently small, the theory will remain in the weak coupling limit, making the Higgs-dilaton scenario self-consistent.

Let us start by considering the inflationary period, characterized by $\xi_\chi\bar{\chi}^2 \ll \xi_h\bar{h}^2$. As shown in Fig. 3.1 the lowest cut-off in this region is the one associated with the gauge interactions Λ_G. The typical momenta of the scalar perturbations produced during inflation are of the order of the Hubble parameter at that time. This quantity can be easily estimated in the Einstein frame, where it is basically determined by the energy stored in the inflationary potential (2.20). We obtain $\tilde{H} \sim \sqrt{\lambda}M_P/\xi_h$. When transformed to the Jordan frame ($H = \Omega\tilde{H}$) this quantity becomes $H \sim \sqrt{\frac{\lambda}{\xi_h}}\bar{h}$, which is significantly below the cut-off scale Λ_G in that region. The same conclusion is obtained for the total energy density, which terns out to be much smaller than Λ_G^4. Moreover, the cut-off Λ_G exceeds the masses of all particles in the Higgs and dilaton background, allowing a self-consistent estimate of radiative corrections (see below).

After the end of inflation the field θ starts to oscillate around the minimum of the potential with a decreasing amplitude due to the expansion of the Universe and due to particle production. This amplitude varies between $M_0/\sqrt{\xi_h}$ and $\sqrt{\xi_\chi}M_0/\xi_h$, where $M_0 = \sqrt{\xi_\chi}\bar{\chi}$ is the asymptotic Planck scale in the low energy regime. As shown in Fig. 2.1, the curvature of the Higgs-dilaton potential around the minimum coincides (up to corrections of $\mathcal{O}(\xi_\chi)$) with that of the Higgs-inflation scenario. All the relevant physical scales, including the effective gauge and fermion masses, agree, up to small corrections, with those of Higgs-inflation. This allows us to directly apply the results of [4, 5, 26] to the Higgs-dilaton scenario. According to these works, the typical momenta of the gauge bosons produced at the minimum of the potential in the Einstein frame is of order $\tilde{k} \sim (\tilde{m}_A/M)^{2/3}M$, with $\tilde{m}_A$ the mass of the gauge bosons in the Einstein frame and $M = \sqrt{\lambda/3}M_P/\xi_h$ the curvature of the potential around the minimum. After transforming to the Jordan frame we obtain $k \sim \left(\frac{\lambda g^4}{48\xi_h}\right)^{1/6}\Lambda_G$, with g the weak coupling constant. The typical momenta of the created gauge bosons are therefore parametrically below the gauge cut-off scale (3.19) in that region.

At the end of the reheating period the system settles down to the minimum of the potential (2.4). In that region the effective Planck mass coincides with the value M_0. The cut-off scale becomes $\Lambda_1 \simeq \sqrt{\xi_\chi}\bar{\chi}/\xi_h \simeq M_P/\xi_h$, since $\xi_\chi\bar{\chi}^2 \equiv M_P^2 \gg \xi_h\bar{h}^2$. This value is much higher than the electroweak scale $m^2 \sim 2\alpha/\xi_\chi M_P$ (cf. Eq. (2.5)) where all the physical processes take place. We conclude therefore that perturbative unitarity is maintained for description of all relevant processes during the whole evolution of the Universe.

Chapter 4

Quantum corrections

In this section we concentrate on the quantum corrections which are generated by the loop expansion and on their influence on the predictions of the model.

Our strategy is as follows. We regularize the quantum theory in such a way that all multi-loop diagrams are finite, whereas the exact symmetries of the classical action (gauge and diffeomorphism transformations as well as scale invariance) are intact. Moreover, we require that the approximate shift symmetry of the dilaton field in the Jordan frame, associated with the smallness of the parameters α, β and ξ_χ and discussed in Section 2, is respected by the regularization. Then we add to the classical action (2.9) an infinite number of counter-terms (including the finite parts as well) which remove all the divergences from the theory and do not spoil the exact and approximate symmetries of the classical action. Since the theory is not renormalizable, these counter-terms will have a different structure from that of the classical action. In particular, terms that are non-analytic with respect to scalar fields will appear [50]. They can be considered as higher-dimensional operators, suppressed by the field-dependent cut-offs. For consistency with the analysis made earlier in this work, we demand that these cut-offs exceed those found in Section 3.

An example of the subtraction procedure which satisfies all the requirements formulated above has been constructed in [54] (see also earlier discussion in [51]). It is based on dimensional regularization in which the t'Hooft-Veltman normalization point μ is replaced by some combination of the scalar fields with an appropriate dimension, $\mu^2 \to F(\chi, h)$ (we underline that we use the Jordan frame here for all definitions). The infinite part of counter-terms is defined as in $\overline{MS}$ prescription, i.e. by subtracting the pole terms in ϵ, where the dimensionality of space-time is $D = 4 - 2\epsilon$. The finite part of counter-terms has the same operator structure as the infinite part, including the parametric dependence on the coupling constants. The requirement of the structure of higher-dimensional operators, formulated in the previous paragraph, puts an important constraint on the choice of the function $F(\chi, h)$, as it is this combination which appears in the denominator of counter-terms [54, 50]. The simplest (but not unique) choice is $F(\chi, h) = \xi_\chi \chi^2 + \xi_h h^2$, identifying the normalization point with the gravity cut-off (3.7) (this corresponds to GR-SI prescription of [54]).

In what follows we will use this "minimal" setup for analysis of the radiative corrections. It will be more convenient to work from now on in the Einstein frame (2.18), where the coupling to gravity is minimal and all non-linearities are moved to the SM sector. First, we will consider the effective action for the background fields corresponding to the "early universe", including inflation and reheating, and then will turn to the accelerating "late universe".

4.1 Early Universe

The inflationary period of the Universe expansion corresponds to background field values (in the Jordan frame) $\xi_\chi \chi^2 \ll \xi_h h^2$, whereas the reheating ends at $\xi_\chi \chi^2 \sim \xi_h^2 h^2$. The initial value of the dilaton field has to be sufficiently large, to keep its present contribution to DE at the appropriate level [28]. The latter fact allows us to neglect all contributions to effective action proportional to Λ_0 in the early universe, stemming from $\tilde{U}_{\Lambda_0}$ given by (2.20). As a result, all corrections due to the dilaton will emerge from its non-canonical kinetic term, whereas all corrections due to Higgs field will emerge from its potential.

The construction of the effective action for the theory is most easily done in the following way: expand the action (2.18) near the constant background of the dilaton and the Higgs fields and drop the linear terms in perturbations. After that, compute all the vacuum diagrams to account for the potential-type corrections and all the diagrams with external legs to account for the kinetic-type corrections.

4.1.1 Dilaton

Let us consider first the quantum corrections to the dilaton itself. Since our subtraction procedure respects the symmetries of the classical action (scale invariance in particular, corresponding to the shift symmetry of the dilaton field r in the Einstein frame), no potential terms for the dilaton can be generated. Thus, the loop expansion can only create two types of contributions, both stemming from its kinetic term . The first type are corrections to the propagator of the field, and as we will show below they are effectively controlled by $(m_H/M_P)^{2k}$, with $m_H^2 \equiv \tilde{U}''(\theta)$ and k the number of loops under consideration. The second type are operators with more derivatives of the field suppressed by appropriate powers of the cut-off associated to the scalar sector of the theory, i.e. M_P. One should bear in mind that the appearance of these operators to the effective action is expected and consistent with the analysis made in the previous section. Their presence does not affect the dynamics of the model, since the scalar cut-off is much larger than the characteristic momenta of the particles involved in all physical processes throughout the whole history of the Universe.

To demonstrate explicitly what we described above, let us consider some of the associated diagrams. Following the ideas of [54], we perform the computations in dimensional regularization in $D = 4 - 2\epsilon$ dimensions. We avoid therefore the use of other regularizations schemes, such as cut-off regularization, where the scale invariance of the theory is badly broken at tree level[1]. The structure of the corrections becomes apparent already from the one-loop order

$$\sim \left(\frac{1}{\epsilon}+f\right) c^{d_1}_{1,1}(\bar{\theta}) \left(\frac{m_H}{M_P}\right)^2 (\partial r)^2 \ ,$$

$$\sim \left(\frac{1}{\epsilon}+f'\right) c^{d_2}_{1,2}(\bar{\theta}) \left[\left(\frac{m_H}{M_P}\right)^2 + \left(\frac{\partial}{M_P}\right)^2\right] (\partial r)^2 \ ,$$

where we represent the dilaton (Higgs) with a dashed (solid) line. To keep the expressions short we denoted with f and f' the finite parts of the diagrams, whose values depend on the normalization point μ and cannot be determined a priori. We also absorbed numerical factors into the background-dependent coefficients $c^{d_i}_{k,V}(\bar{\theta})$, which are combinations of hyperbolic functions. Their form is complicated because of the non-canonically normalized kinetic term of the dilaton and it depends on the particular diagram d_i under consideration, the number of loops k, as well as the number of vertices V[2]. Their values are always smaller than unity, and vary slightly with the value of the angular variable $\bar{\theta}$.

In two-loops the situation is somehow similar. The divergent (and finite) part of the corrections (consider for example the diagrams presented in Fig. 4.1) is proportional to

$$c^{d_i}_{2,V}(\bar{\theta}) \left[\left(\frac{m_H}{M_P}\right)^4 + \left(\frac{m_H}{M_P}\right)^2 \left(\frac{\partial}{M_P}\right)^2 + \left(\frac{\partial}{M_P}\right)^4\right] (\partial r)^2 \ , \quad V \leq 4 \ . \tag{4.1}$$

Notice that the operator with two powers of derivatives that is present already from the first order, reappears suppressed by $(m_H/M_P)^2$ in addition to the cut-off of the theory.

It is not difficult to convince oneself that this happens in the higher order diagrams as well. The structure of the corrections is therefore proportional to

$$c^{d_i}_{k,V}(\bar{\theta}) \left[\left(\frac{m_H}{M_P}\right)^{2k} + \left(\frac{m_H}{M_P}\right)^{2k-2} \left(\frac{\partial}{M_P}\right)^2 + \ldots + \left(\frac{m_H}{M_P}\right)^2 \left(\frac{\partial}{M_P}\right)^{2k-2} + \left(\frac{\partial}{M_P}\right)^{2k}\right] (\partial r)^2 \ . \tag{4.2}$$

Note that the corrections from diagrams with gauge bosons and fermions running inside the loops are given also by (4.2), by consistently replacing m_H with the mass of the particle considered.

[1]Similar arguments about the artifacts created by regularization methods that explicitly break scale invariance can be found for instance in Ref. [52].

[2]We introduce the index d_i to distinguish between the diagrams with the same number of vertices but different combinations of hyperbolic functions that appear in higher loops.

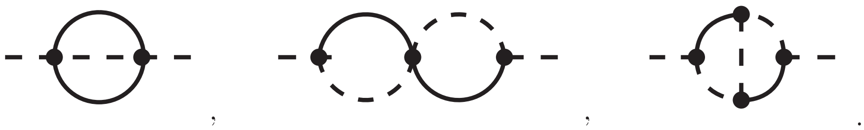


Figure 4.1: Some of the two-loop diagrams for the dilaton.

4.1.2 Higgs

We now turn to the corrections to the Higgs field. Once again we consider first the potential-type contributions. The situation now is more complicated, since the effective potential for the Higgs field θ will be modified by terms stemming from the scale-invariant part of the tree-level potential as well as from the non-canonical kinetic term of the dilaton field r, with the latter starting from the second order in perturbation theory.

To keep the notation as simple as possible, we express the scale-invariant part of the potential given by eq. (2.20) in the following compact form

$$\tilde{U}(\theta) = U_0 \left(u_0 + \sum_{n=1}^{2} u_n \cosh[2na\theta/M_P] \right) , \quad U_0 = \frac{M_P^4}{4\xi_h^2(1-\varsigma)^2} , \tag{4.3}$$

where

$$u_0 = c^2 - c\sigma + \frac{3\sigma^2}{8} + \frac{3\beta'}{2} , \; u_1 = \frac{\sigma^2}{2} - c\sigma - 2\beta' , \; u_2 = \frac{\sigma^2}{8} + \frac{\beta'}{2} , \quad \text{with} \quad \beta' \equiv \frac{\beta}{\lambda} \left(\frac{1+6\xi_h}{1+6\xi_\chi} \right)^2 . \tag{4.4}$$

We expand the field around its background value, so the above yields

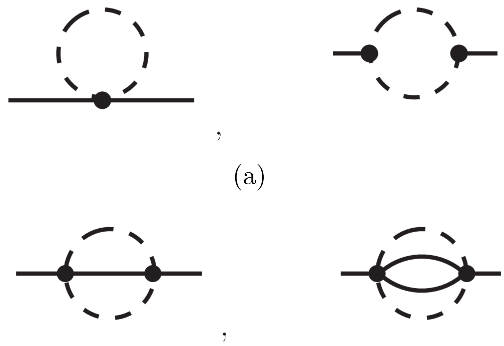
$$\begin{aligned} \lambda\tilde{U}(\bar{\theta}+\delta\theta) &= \lambda U_0 \sum_{n=1}^{2} u_n \sum_{l=0}^{\infty} \frac{\cosh^{(l)}[2na\bar{\theta}/M_P]}{l!} \left(\frac{2na\delta\theta}{M_P} \right)^l \\ &= \lambda U_0 \sum_{n=1}^{2} \sum_{l=0}^{\infty} u_n \left[c_{n,l} \cosh[2na\bar{\theta}/M_P] \left(\frac{a\delta\theta}{M_P} \right)^{2l} + d_{n,l} \sinh[2na\bar{\theta}/M_P] \left(\frac{a\delta\theta}{M_P} \right)^{2l+1} \right] , \end{aligned} \tag{4.5}$$

where $c_{n,l}$ and $d_{n,l}$ account for numerical coefficients and combinatorial factors. Since the theory is non-renormalizable, the perturbative expansion creates terms which do not have the same background dependence of the original potential. The contributions turn out to be of the form[3]

$$\frac{\lambda^{i+j} M_P^4}{[4\xi_h^2(1-\varsigma)^2]^{i+j}} \left[g\left(\frac{1}{\epsilon}\right) + f_{i,j} \right] \sum_{n,m} u_n^i u_m^j \cosh^i[2na\bar{\theta}/M_P] \sinh^j[2ma\bar{\theta}/M_P] , \tag{4.6}$$

where we have left aside numerical factors, denoted with $f_{i,j}$ the (finite) integration constant, and $g(1/\epsilon)$ is a function of the divergent terms. Note that if we set $\beta = 0$, we

[3] We choose not to express the corrections in terms of m_H/M_P, because here the combinations of the hyperbolic function are much simpler than the ones that appear from the dilaton's kinetic term.



(a)

(b)

Figure 4.2: Characteristic diagrams coming from the non-canonical kinetic term of the dilaton field r. The dashed (solid) line represents the dilaton (Higgs). From the one-loop diagrams in (a), the first one vanishes in dimensional regularization since the dilaton is massless, whereas the second contributes only terms with more than two derivatives of the Higgs. In (b) we consider two and three loop diagrams which, apart from generating higher dimensional operators, contribute to the effective potential once we amputate them.

make sure that terms which contribute to the cosmological constant given by Eq. (2.6) will not be generated by the loop expansion.

By inspection of the structure of divergences, we can see that the leading corrections are those appearing with the lowest power in ς. To gain insight on their contribution, we calculate the finite part of Eq. (4.6) when the hyperbolic functions take their maximal values, which happens for $\theta_{\max} = \theta_0 \equiv M_P/\alpha \tanh^{-1}[\sqrt{1-\varsigma}]$. Then we get

$$\frac{\lambda^{i+j}}{[4\xi_h^2(1-\varsigma)^2]^{i+j}} f_{i,j} \sum_{n,m} u_n^i u_m^j \cosh^i[2na\bar{\theta}/M_P] \sinh^j[2ma\bar{\theta}/M_P]\Big|_{\bar{\theta}=\theta_{\max}} \sim \left(\frac{\lambda\varsigma}{4\xi_h^2}\right)^{i+j} f_{i,j} \ , \tag{4.7}$$

making the corrections coming from the order $i+j+1$ negligible compared to the ones from $i+j$ order. In the last step we have set $c=1$, $\sigma=\varsigma$, which, given the small value of the parameter α appearing in (2.22), constitutes a very good approximation.

As we mentioned earlier, potential-type corrections to the Higgs field are also generated from diagrams associated to the kinetic term of the dilaton r, starting from two loops. This happens because the first order vacuum diagrams with dilaton running in the loop, vanish. If we consider higher loop diagrams, like those in fig. 4.2(b) but without momenta in the external legs, we see that even though the background dependence of the corrections is complicated due to the non-canonically normalized dilaton that runs inside the loops, their contributions to the effective action are of the same order as those in eq. (4.7).

We now turn to the kinetic-type corrections to the Higgs field. By that we mean corrections to the propagator of the Higgs, as well as terms with more derivatives of the field suppressed by the scalar cut-off. The first type of contributions come only from the scale-invariant part of the potential given by (4.3) , when the momenta associated to the external legs are considered. It is not difficult to show that these are precisely of the same form as the ones in (4.6). The second type of contributions, i.e. the higher dimensional operators, are generated both from the potential of the Higgs in higher loops, as well as from the non-vanishing diagrams associated to the non-canonical kinetic term of the dilaton. The terms we get are proportional to

$$\frac{\partial^2}{M_P^2}(\partial\theta)^2 \ , \quad \frac{\partial^4}{M_P^4}(\partial\theta)^2 \ \ldots \tag{4.8}$$

so they can be safely neglected for the typical momenta in the different epochs of the evolution of the Universe.

Before moving on, we would like to comment on the appearance of mixing terms with derivatives of the fields. These manifest themselves when we consider diagrams with both fields in the external legs. They are higher dimensional operators, and it can be shown that they appear suppressed by the scalar cut-off of the theory, as was the case before.

Since the kinetic-type operators do not modify the dynamics of the model, to understand how the predictions of the model are altered it is sufficient to consider the potential-type corrections. In one-loop, it is straightforward to show that [53]

$$\begin{aligned}\Delta\tilde{U} \simeq\ & \frac{\lambda^2}{64\pi^2}\frac{a^4 M_P^4}{4\xi_h^4(1-\varsigma)^4}\left(\frac{1}{\bar{\epsilon}}+f_{2,0}\right)\Bigg[\varsigma^2\left(\frac{1+\cosh[4a\bar{\theta}/M_P]}{2}\right)\\ & -\varsigma^3\left(\frac{1+\cosh[4a\bar{\theta}/M_P]}{2}+\cosh[2a\bar{\theta}/M_P]\cosh[4a\bar{\theta}/M_P]\right)\\ & +\varsigma^4\left(\frac{1+\cosh[4a\bar{\theta}/M_P]}{2}+2\cosh[2a\bar{\theta}/M_P]\cosh[4a\bar{\theta}/M_P]+\frac{1+\cosh[8a\bar{\theta}/M_P]}{2}\right)\Bigg] \ ,\end{aligned} \tag{4.9}$$

where we sorted the corrections by increasing powers of ς, and defined $1/\bar{\epsilon} = 1/\epsilon - \gamma + \log 4\pi$ for convenience.

For the Higgs-dilaton model to be viable, scale-invariance as well as the asymptotic shift symmetry should be preserved at the level of the UV-complete theory. It is therefore necessary to use the "SI-prescription" formulated in [54], which is practically implemented by replacing the renormalization point μ with a field-dependent effective cut-off. Even though the following result is rather insensitive to the choice of the renormalization point, in our calculation we identified μ as the Planck mass, which corresponds to the renormalization scheme I in [2, 7]. This is of course not a unique choice, since the requirement for SI cannot fix the details of the renormalization prescription, but it is in a way favoured since it has the same form of the non-minimal coupling of the fields to the scalar curvature. With this choice, the finite constant $f_{2,0}$ will be given by

$$\begin{aligned} f_{2,0} &= \frac{3}{2} - \log\left[\frac{\tilde{U}''(\bar{\theta})}{\mu^2}\right]_{\mu=M_P} \\ &= \frac{3}{2} - \log\left[\frac{\lambda a^2}{2\xi_h^2(1-\varsigma)^2}\left[-2\varsigma\cosh[2a\bar{\theta}/M_P] + \varsigma^2(\cosh[2a\bar{\theta}/M_P] + \cosh[4a\bar{\theta}/M_P])\right]\right] . \end{aligned} \tag{4.10}$$

If we use the $\overline{MS}$ scheme, the remaining (logarithmic) corrections will be suppressed by an overall factor of the order of $\mathcal{O}(10^{-15})$ as well as powers of ς, making the contribution negligible and allowing us to approximate the value of θ at the end of inflation by its classical value $\theta_{\rm f} \simeq 0$.

Taking into account the one-loop correction to the effective potential the tilt n_s of the primordial scalar spectrum becomes

$$n_s(k_0) - 1 \simeq -8\xi_\chi + \frac{\lambda\xi_\chi^2}{96\pi^2\xi_h^2} f_{2,0} , \quad \text{for} \quad 1 \lesssim 4\xi_\chi N^* \ll 4N^* , \tag{4.11}$$

and therefore hardly modified compared to the one given by Eq. (2.25), calculated only with the tree-level potential.

4.2 Late Universe

When the reheating period of the Universe ends, the system is outside the scale-invariant region and the field θ has almost settled down at the minimum of the potential. i.e. $\tanh^2[a\theta/M_P] \simeq 1-\sigma$. As a consequence, there is only one almost massless degree of freedom, the physical dilaton r. From Eqs. (2.18) and (2.21) we see that the action is

$$\mathscr{L}_{\rm DE} \simeq -\frac{(\partial r)^2}{2} - \frac{\Lambda_0'}{\gamma^2}\varsigma^2 e^{-4\gamma r/M_P} , \tag{4.12}$$

where we absorbed the contributions of the hyperbolic functions associated to the value of θ field at the minimum of the potential into the redefinition of Λ_0. From the expansion of the potential around the background, it is clear that all the contributions to the effective action will once again be suppressed by powers of the exponent $e^{-\gamma\bar{r}/M_P}$ in addition to powers of M_P, and therefore will not affect the predictions of the model concerning the dark energy equation of state parameter given by Eq. (2.27). Taking into account the results of Sections 4.1 and 4.2 we conclude the quantum corrections do not modify the classical consistency relation (2.29) characterizing Higgs-dilaton cosmology.

Chapter 5

Conclusions

The purpose of this easy was to study the self-consistency of the Higgs-dilaton cosmological model. We determined the field-dependent UV cut-offs and studied their evolution in the different epochs throughout the history of the Universe. We showed that the cut-off value is higher than the relevant energy scales in the different periods, making the model a viable effective field theory describing inflation, reheating, and late-time acceleration of the Universe.

Since the theory is non-renormalizable, the loop expansion creates an infinite number of divergences, something that may challenge the classical predictions of the Higgs-dilaton model. We showed that this is not the case if the UV-completion of the theory respects scale-invariance and the approximate shift symmetry for the dilaton field. We used the "SI-prescription" of [54], with a field-dependent renormalization point which in the Einstein frame coincides with M_P and concluded that the most important contribution to the theory comes from the one-loop corrections to the effective potential. This modification leaves practically intact the relation which allows us to connect the tilt of the scalar spectrum to the deviation of the dark energy from a cosmological constant.

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