Simulating simplified versions of the IKKT matrix model*

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We simulate a supersymmetric matrix model obtained from dimensional reduction of 4d SU(N) super Yang-Mills theory (a 4d counter part of the IKKT model or IIB matrix model). The eigenvalue distribution determines the space structure. The measurement of Wilson loop correlators reveals a universal large N scaling. Eguchi-Kawai equivalence may hold in a finite range of scale, which is also true for the bosonic case. We finally report on simulations of a low energy approximation of the 10d IKKT model, where we omit the phase of the Pfaffian and look for evidence for a spontaneous Lorentz symmetry breaking.

1. MOTIVATION

Several candidates for a constructive definition of superstring theory have recently attracted attention. Here we focus on the IKKT model (or IIB matrix model) [1], which is supposed to correspond to IIB superstring theory in the large N limit. That correspondence is supported by some analytical arguments. This matrix model is formally obtained from ordinary super YM gauge theory in the zero volume limit (one point).

Here we study directly the large N dynamics of large N reduced matrix models. Some results were obtained before for the "bosonic case" (where the fermions are dropped by hand), but we now want to address mainly the SUSY case. In particular we simulate the 4d counterpart of the IKKT model [2] — we denote it as 4d IIB matrix model — which corresponds again to the dimensional reduction of 4d super YM gauge theory. This model has also been studied analytically [3] and numerically in the framework of dynamical triangularization [4]. Here we report on direct Monte Carlo simulations using the Hybrid R algorithm [5]. Conclusive results can be obtained because in the 4d version the fermion determinant is real positive — in contrast to the 10d IKKT model, where simulations would be plagued by a sign problem. We got away with a computational effort of $O(N^5)$ in the SUSY case, and of $O(N^3)$

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in the bosonic case.

2. THE 4D II-B MATRIX MODEL

The 4d IIB matrix model is given by

$$
\begin{aligned}
\mathbf{Z} &= \int dA \, e^{-S_b} \int d\bar{\psi} d\psi \, e^{-S_f} \\
S_b &= -\frac{1}{4g^2} \operatorname{Tr}[A_\mu, A_\nu]^2 \\
S_f &= -\frac{1}{g^2} \operatorname{Tr}(\bar{\psi}_\alpha \Gamma^\mu_{\alpha\beta}[A_\mu, \psi_\beta])
\end{aligned} \tag{1}
$$

where A_μ, $\bar{\psi}_\alpha$, ψ_α ($\mu = 1 \ldots 4$, $\alpha = 1, 2$) are complex, traceless $N \times N$ matrices, and the A_μ (only) are Hermitean. We use $\vec{\Gamma} = i\vec{\sigma}$, $\Gamma_4 = \mathbb{1}$. In addition to SUSY and SO(4) invariance, this model has a SU(N) symmetry, which is inherited from gauge invariance.

The first question about this model is if it is well-defined as it stands. Since the integration domain of dA is non-compact, divergences are conceivable. However, our results, as well as results on a number of special cases [6–8] confirm consistently that this model *is* well-defined at any N; there is no need to impose a IR cutoff. This implies that the only parameter g is simply a *scale parameter*, rather than a coupling constant. It can be absorbed by introducing dimensionless quantities. The challenge is, however, to tune g as a function of N so that the correlators are finite at $N \to \infty$, see Sections 4 and 5.

3. THE SPACE STRUCTURE

In the IIB matrix model, the space coordinates arise dynamically from the eigenvalues of the matrices A_μ [1]. In general the latter cannot be diagonalized simultaneously, which implies that we deal with a non-classical space. We measure its uncertainty by

$$\Delta^2 = \frac{1}{N}\left[\mathrm{Tr}(A_\mu^2) - \max_{U \in SU(N)}\sum_i\{(UA_\mu U^\dagger)_{ii}\}^2\right],$$

and the "maximizing" matrix U is also used for introducing the coordinates of N points,

$$x_{i,\mu} = (U_{max}A_\mu U_{max}^\dagger)_{ii} \qquad (i = 1\ldots N). \qquad (2)$$

What we are really interested in is their pairwise separation $r(x_i, x_j) = |x_i - x_j|$, and we show the distribution $\rho(r)$ in Fig. 1. We observe $\rho \approx 0$ at short distances $(r/\sqrt{g} \lesssim 1.5)$, hence a UV cutoff is generated *dynamically*. We also see that increasing N favors larger values of r. To quantify this effect we measure the "extent of space" [2]

$$R_{new} = \int_0^\infty r\rho(r)\, dr. \qquad (3)$$

Fig. 2 shows R_{new} and Δ as functions of N (at $g = 1$). The inclusion of fermions enhances R_{new} and suppresses Δ, keeping their product approximately constant. The lines show that both quantities follow the same power law, R_{new}, $\Delta \propto N^{1/4}$, in SUSY and in the bosonic case. In SUSY this behavior is consistent with the branched polymer picture: there one would relate the number of points as $N \sim (R_{new}/\ell)^{d_H}$, where ℓ is some minimal bond, which corresponds to the above UV cutoff. The Hausdorff dimension $d_H = 4$ then reveals consistency with our result.

4. POLYAKOV AND WILSON LOOPS

We define the Polyakov loop P and the Wilson loop W — which is conjectured to correspond to the string creation operator — as

$$P(p) = \frac{1}{N}\mathrm{Tr}\left(e^{ipA_1}\right), \qquad (4)$$

$$W(p) = \frac{1}{N}\mathrm{Tr}\left(e^{ipA_1}e^{ipA_2}e^{-ipA_1}e^{-ipA_2}\right). \qquad (5)$$

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[2]Note that the quantity $R^2 = \int_0^\infty r^2\rho(r)\, dr$ diverges, since $\rho(r) \propto r^{-3}$ at large r (see second Ref. in [6]).
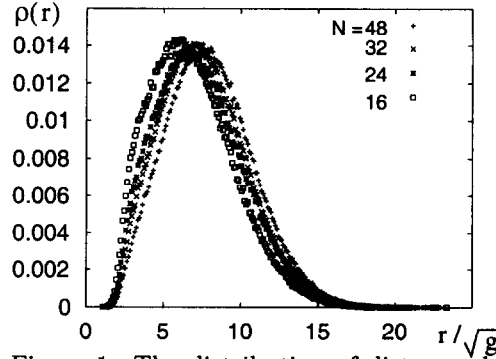

Figure 1. The distribution of distances between space-points in the SUSY case at various N.
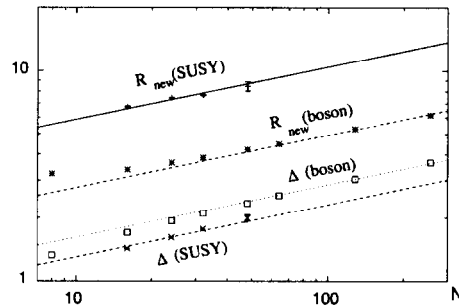

Figure 2. The "extent of space" R_{new} and the space uncertainty Δ as functions of N at $g = 1$.

Of course the choice of the components of A_μ is irrelevant, and the parameter $p \in \mathbb{R}$ can be considered as a "momentum".

Now $g(N)$ has to be tuned so that $\langle P\rangle$, $\langle W\rangle$ remain finite as $N \to \infty$. This is achieved by

$$g \propto 1/\sqrt{N}, \qquad (6)$$

which leads to a beautiful large N scaling; Fig. 3 shows the invariance of $\langle P\rangle$ for $N = 16\ldots 48$ in SUSY. Also the bosonic case scales accurately [2].
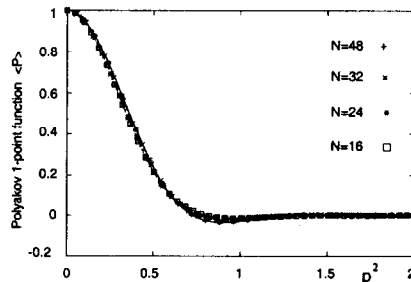

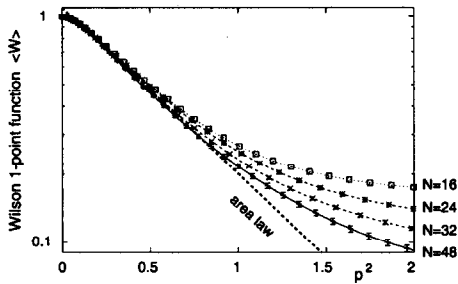Figure 3. The Polyakov function in the SUSY case for various values of N and $g^2 N = const$.

Figure 4. The Wilson loop in the SUSY case for various values of N and $g^2 N = const.$

The historic 2d Eguchi-Kawai model [9] had a $(\mathbb{Z}_N)^2$ symmetry, which implied $\langle P(p \neq 0) \rangle = 0$, a property which was crucial for the proof of the Eguchi-Kawai equivalence to gauge theory. As we see, this property is not fulfilled here, but $\langle P(p) \rangle$ falls off rapidly, towards a regime where the assumption of this proof holds approximately.

We proceed to a more explicit test of Eguchi-Kawai equivalence by checking the area law for $\langle W(p) \rangle$. Fig. 4 shows that the area law seems to hold in a finite range of scale. Remarkably, the behavior is very similar [2] in the bosonic case. There we further investigated the behavior at much larger N [10], and we observed that the power law regime does neither shrink to zero — as it was generally expected — nor extend to infinity — a scenario which seems possible from Fig. 4. At least in the bosonic case its range remains finite at large N. Recently the same behavior was observed in a study of the 10d bosonic case [11].

5. MULTIPOINT FUNCTIONS

We now consider connected multipoint functions $\langle \mathcal{O}_1 \mathcal{O}_2 \dots \mathcal{O}_n \rangle_{con}$, $\mathcal{O}_i$ being a Polyakov or a Wilson loop. We wonder if it is possible to renormalize all of those multipoint functions simply by inserting $\mathcal{O}_i^{(ren)} = Z \mathcal{O}_i$, so that a single factor Z renders all functions $\langle \mathcal{O}_1^{(ren)} \mathcal{O}_2^{(ren)} \dots \mathcal{O}_n^{(ren)} \rangle_{con}$ (simultaneously) finite at large N.

It turns out that such a universal renormalization factor seems to exist in SUSY. We have to set again $g \propto 1/\sqrt{N}$, and then $Z \propto N$ provides large N scaling, as we observed for a set of 2, 3 and 4-point functions. Two examples are shown in Fig. 5. Our observation can be summarized by the SUSY rule

$$\langle \mathcal{O} \rangle = O(1) \ , \quad \langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = O(N^{-n}) \quad (n \geq 2).$$
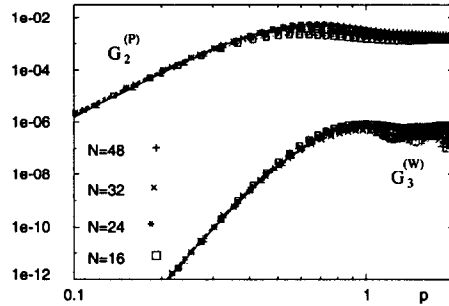

Figure 5. A Polyakov 2-point function $(G_2^{(P)} = \langle [\mathrm{Im}P(k)]^2 \rangle)$ and a Wilson 3-point function $(G_3^{(W)} = \langle [\mathrm{Im}W(k)]^2 \mathrm{Re}W(k) \rangle - \langle [\mathrm{Im}W(k)]^2 \rangle \langle \mathrm{Re}W(k) \rangle)$ (both connected), with $g^2 N = const.$ and renormalization factor $Z \propto N$, which leads to large N scaling in the SUSY case.

This implies that large N factorization holds, $\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = \langle \mathcal{O}_1 \rangle \dots \langle \mathcal{O}_n \rangle + O(N^{-2})$, as in gauge theory, although coupling expansions are not applicable here.

For the bosonic case, a $1/d$ expansion [7] suggests large N factorization to hold as well, but it also predicts $\langle \mathcal{O}_1 \dots \mathcal{O}_n \rangle = O(N^{-2(n-1)})$ $(n \geq 2)$. This is confirmed numerically [2]: in particular the 3-point functions shown in Fig. 5 now requires $Z^3 \propto N^4$. Therefore no universal renormalization factor Z exists in the bosonic case, which is an important qualitative difference from the SUSY case.

6. SIMULATIONS IN 10 DIMENSIONS

We also performed simulations in $d = 10$ [12], where we simplified the IKKT model as follows:

(1) We use a 1-loop approximation, which is expected to capture the low energy dynamics. This amounts to an effective action, keeping track of off-diagonal elements only to the quadratic order, in the spirit of Ref. [8].

(2) We omit the phase of the Pfaffian by hand, in order to avoid the sign problem. Thus a Monte Carlo study becomes feasible.

The validity of (1) is supported by our results for R_{new}, but (2) is certainly a drastic step. Still one could hope to observe basic properties of the IKKT model at least qualitatively. These simplifications allow for a simulation effort of only $O(N^3)$.

Our main interest here is if the eigenvalue dis-

tribution of the A_μ indicates a spontaneous symmetry breaking (SSB) of SO(10) invariance, as it was suggested in the formulation of the IKKT model [1].

To this end, we consider the moment of inertia

$$T_{\mu\nu} = \frac{2}{N(N-1)} \sum_{i>j} (x_{i\mu} - x_{j\mu})(x_{i\nu} - x_{j\nu}) \quad (7)$$

($i = 1 \ldots N$), and we measure its 10 eigenvalues. A gap in this spectrum would indicate the SSB of Lorentz symmetry. However, this cannot be observed, even though we raised N up to 512. On the contrary, we observe that the eigenvalue distribution becomes more and more isotropic as N increases, see Fig. 6, and the same is true in $d = 6$ [12]. We conclude that if SSB of Lorentz symmetry occurs in the IKKT model, then it must be driven by the imaginary part of the action.
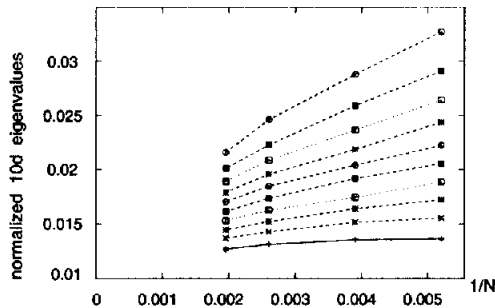

Figure 6. The spectrum of the moment of inertia (normalized by $\sqrt{N}$) for $N = 192, 256, 384, 512$.

7. CONCLUSIONS

We first simulated the 4d IIB matrix model, both, SUSY and bosonic. We varied N up to 48, which turned out to be sufficient to study the large N dynamics.

We confirmed that the model is well-defined as it stands, hence g is a pure scale parameter. The space coordinates arise from eigenvalues of the bosonic matrices A_μ. The extent of space follows a power law. In SUSY this agrees with the branched polymer picture. Fermions leave the power unchanged but reduce the space uncertainty — though it remains finite at large N.

The large N scaling of Polyakov and Wilson loops and their correlators requires $g \propto 1/\sqrt{N}$ in SUSY and in the bosonic case, but the wave

function renormalization is qualitatively different: only in SUSY a universal renormalization exists.

The area law for Wilson loops holds in a finite range of scale for the SUSY and the bosonic case. The latter comes as a surprise, and we checked up to rather large N that this range remains indeed finite. Hence Eguchi-Kawai equivalence to ordinary gauge theory [9] may hold in some regime.

Finally we simulated a 10d low energy effective theory, where the phase was dropped by hand. We could not observe any sign of a spontaneous breaking of Lorentz symmetry.

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