ELSEVIER

Nuclear Physics B (Proc. Suppl.) 63A-C (1998) 748–750

NUCLEAR PHYSICS B
PROCEEDINGS
SUPPLEMENTS

Intrinsic Geometric Structure of $c = -2$ Quantum Gravity

J.Ambjørn[a] , K.N.Anagnostopoulos[a], T.Ichihara[b] , **L.Jensen**[a], N.Kawamoto[c] , Y.Watabiki[b] and K.Yotsuji[c]

[a]Niels Bohr Institute, Blegdamsvej 17
2100 Copenhagen Ø, Denmark

[b]Tokyo Institute of Technology, O-okayama
Meguro, Tokyo, Japan

[c]Department of Physics, Hokkaido University
Sapporo, Japan

We couple $c = -2$ matter to 2-dimensional gravity within the framework of dynamical triangulations. We use a very fast algorithm, special to the $c = -2$ case, in order to test scaling of correlation functions defined in terms of geodesic distance and we determine the fractal dimension d_H with high accuracy. We find $d_H = 3.58(4)$, consistent with a prediction coming from the study of diffusion in the context of Liouville theory, and that the quantum space–time possesses the same fractal properties at all distance scales similarly to the case of pure gravity.

1. The model

Our starting point is the partition function of Euclidean 2d quantum gravity at fixed volume and discretize it by using dynamical triangulations with a fixed number of equilateral triangles N. The integral over metrics changes into a sum over triangulations $\mathcal{T}_N$, and by coupling the quantum gravity to c Gaussian fields, the partition function is rewritten as

$$Z_N = \sum_{\mathcal{T}_N} \frac{1}{\mathcal{S}_{\mathcal{T}_N}} (det' C_{\mathcal{T}_N})^{-c/2} \qquad (1)$$

Here $C_{\mathcal{T}_N}$ is the adjacency matrix of the ϕ^3 graph, which is dual to $\mathcal{T}_N$, $\mathcal{S}_{\mathcal{T}_N}$ is a symmetry factor, and $det' C_{\mathcal{T}_N}$ denotes the determinant of $C_{\mathcal{T}_N}$ with the zero-eigenvalue removed. This partition function also serves as the defining equation for coupling to matter with $c < 0$. We restrict ourselves to surfaces with the topology of a sphere, for reasons which will soon be clear. By selecting $c = -2$ it is possible to construct a recursive sampling algorithm for the numerical generation of an ensemble of surfaces [2]. Two results from graph theory are crucial here. First, the determinant (appropriately defined) of $C_{\mathcal{T}_N}$ is equal to the number of spanning trees in the ϕ^3 graph. Second, a graph can be embedded on a sphere if - and only if - it is planar. Thus restricting ourselves to configurations with the topology of a sphere, we can decompose the sum over all triangulations into a sum over all combinations of ϕ^3 trees and rainbows - a rainbow being a set of non-crossing lines which connect the endpoints of the tree-graphs. Both the trees and the rainbows satisfy Schwinger-Dyson equations, and by solving these equations, we can generate the correct ensemble of configurations using a recursive sampling algorithm. The time for generating a configuration grows linearly with its size, a rare case of a non–trivial statistical system with subexponential time growth of statistically completely independent configurations. This fact has enabled us to generate a substantial number of configurations with up to 8 million triangles.

2. Definition of the fractal dimension

The fractal dimension is defined from scaling of correlation functions defined in terms of geodesic distance r, therefore we have to define geodesic distance on the triangulated surfaces. This is done in two ways. Either as the number of links between vertices in the ϕ^3 graph (the dual lat-

PII S0920-5632(97)00892-X

tice), or the number of links between the vertices of the triangles (the direct lattice). It is a non–trivial fact that both definitions are proportional to each other after taking the quantum average. In order to study the intrinsic structure, consider the average volume $n_N(R)$ of a spherical shell at a distance R. We can then define *two* fractal dimensions d_h and d_H in the following way:

$$n_N(r) = N^{1-1/d_H} F_1(x), \quad x = \frac{r+a}{N^{1/d_H}}. \tag{2}$$

where $F_1(x) \sim x^{d_h-1}$ for $x \sim 0$. The existence of the scaling variable x at all length scales is a non–trivial fact to be tested by the simulations. The "shift" a is a finite size correction. If space–time looks the same at all scales then $d_h = d_H$. Our simulations provide evidence, that this is indeed the case. For the correct value of d_H and a, the distributions $n_N(r)$, plotted as a function of x, will fall on top of each other if they actually do scale. That this is indeed the case, and that the finite-size correction a is necessary, can be seen in fig. 1 and fig. 2 were the dual lattice distance has been used and the configuration sizes range from 2k to 8M triangles.

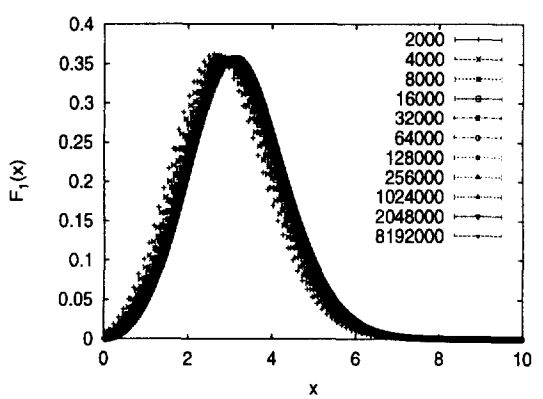

Figure 1. $F_1(x)$ without shift ($a = 0$)

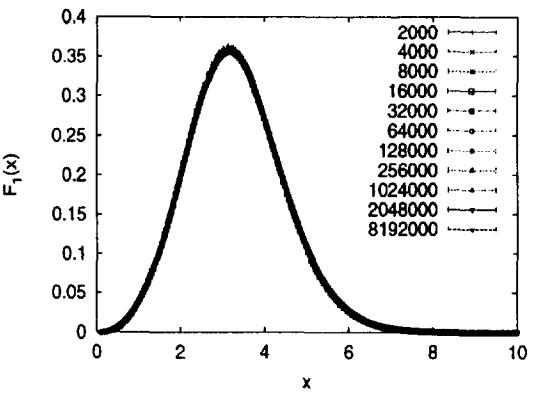

Figure 2. $F_1(x)$ with shift ($a = 4.50$)

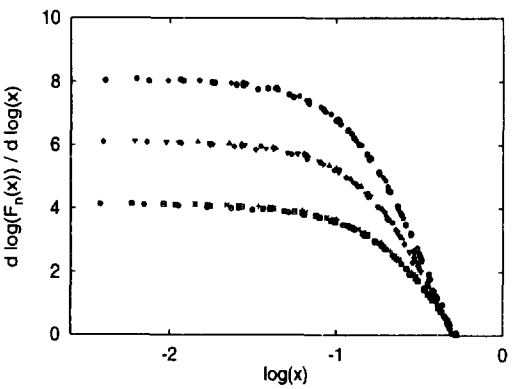

Figure 3. Short distance higher moments

3. Higher moments

We can also consider the higher moments $\langle l^n \rangle_{r,N}$ of the boundary length l. These are defined in terms of the loop–length distribution function $\rho(r,l)$, which counts the number of disconnected loops of the set of points at distance r from a given point, as $\langle l^n \rangle_{r,N} = \int dl\, l^n\, \rho(r,l)$. From Liouville theory we expect $[l^2] = [A]$, but the simulations are consistent with the scaling

$$\langle l^n \rangle_{r,N} = N^{\frac{2n}{d_H}} F_n(x) \quad , n \geq 2\,, \tag{3}$$

where $F_n(x) \sim x^{2n}$ for $x \ll 1$. This implies that $[l^{d_H/2}] = [A]$. The small distance scaling can be seen clearly in fig.3 where we show a plot of the logarithmic derivative of the moments $n = 2, 3, 4$. Trying to collapse the distributions of the mo-

Table 1
The fractal dimension determined from different methods.

		lattice	method
d_h	3.58(5)	direct	Short distance scaling
d_h	3.50(4)	dual	Short distance scaling
d_H	3.60(3)	direct	Collapsing the 2-point function
d_H	3.55(3)	dual	Collapsing the 2-point function
d_H	3.577(8)	direct	Intersection of $R_{a,N}$
d_H	3.574(20)	dual	Intersection of $R_{a,N}$
d_H	3.64(7)	direct	Collapsing of higher moments

ments is consistent with eq.(3).

4. Determination of fractal Dimension

Using different methods to collapse the distributions [1] we can determine both d_H and d_h with great precision. The results using these methods are shown in table 1. $R_{a,N}(d_H) = 1/N^{1+1/dH} \sum_r (r+a) n_N(r)$ is the average radius of the universe [1] and d_H can be determined by tuning d_H and a so that the value of $R_{a,N}(d_H)$ is independent of N. As is shown in fig.4 with the link distance, this works very well indeed. Two conclusions stand out. First, d_h and d_H

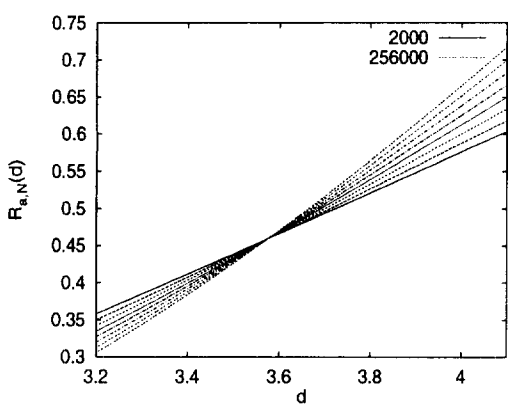

Figure 4. $R_{a,N}$ on the direct lattice.

are equal - i.e. the generated surfaces look the same at small as well as at large distances. This is a non-trivial observation which is not always satisfied, for example in multi-critical branched polymers[1,5]. Then, there exists two different predictions for the value of d_H. One is obtained by using the diffusion equation in Liouville theory[3] which predicts

$$d_H = 2\frac{\sqrt{25-c}+\sqrt{49-c}}{\sqrt{25-c}+\sqrt{1-c}} = 3.561... \qquad (4)$$

whereas redefining the distance in the context of transfer matrix theory[4] gives

$$d_H = \frac{2}{|\gamma(c)|} = \frac{24}{1-c+\sqrt{(25-c)(1-c)}} = 2 \qquad (5)$$

where $\gamma(c)$ is the string susceptibility. Comparing these with the results in table 1, it is apparent that the Liouville-prediction is strongly favored, whereas the prediction from transfer matrix theory is disproved.

REFERENCES

1. J.Ambjørn, K.N.Anagnostopoulos, T.Ichihara, L.Jensen, N.Kawamoto, Y.Watabiki and K.Yotsuji, Phys.Lett. B397(1997) 177; hep-lat/9706009
2. V.A.Kazakov, I.K.Kostov and A.A.Migdal, Phys.Lett 157B (1985) 295; D.V.Boulatov, V.A.Kazakov, I.K.Kostov and A.A.Migdal, Nucl.Phys B275 (1986)641.
3. N.Kawamoto, in Nishinomiya 1992, Proceedings, Quantum Gravity 112, ed. K.Kikkawa and M.Ninomiya (World Scientific); Y.Watabiki, Prog.Theor.Phys.Suppl.114 (1993)1
4. N.Ishibashi and H.Kawai, Phys.Lett. B314 (1993) 190; Phys.Lett. B322 (1994) 67.
5. J.Ambjørn, B.Durhuus and T.Jonsson, Phys.Lett. B244 (1990) 403.